Covering a Wishing Well

Assessment Type	**Recommended Grade Level**	**MYP Criterion Level**
Real-life Application of Mathematics	Grade 8 (MYP3)	MYP 3

MYP Assessment Criteria
Criterion C: Communicating
Criterion D: Applying mathematics in real-life contexts

MYP Command Terms Used
identify, show, use, calculate, summarize, describe, apply, find, discuss, draw, write down, explain, state, suggest, solve

MYP Global Context
Scientific and technical innovation

MYP Key Concept
Form

MYP Related Concept
Space

MYP Branch of Mathematics
Spatial reasoning

MYP Topics and Skills
- Area of plane figures (rectangles) and compound shapes
- Pythagoras' theorem

Prior Knowledge Needed
- Calculating the area of rectangles
- Using Pythagoras' theorem to find length of missing leg

Assessment Description

In this assessment, students work through two different methods of finding the area of a cover for a circular wishing well using only rectangular pieces with as little extra material as possible. Throughout the task, students apply methods of approximation to get increasingly closer to the actual area of a circle, at first by using a square grid (Achievement Levels 1-4) and then by using an increasing number of rectangular strips (Achievement Levels 5-8). Students explain the level of accuracy and whether their method makes sense.

Materials Needed

Pencil, paper, ruler, calculator (required)

Task-specific instructions / Recommendations

The use of calculator is required for this assessment task.

Assessment Criterion C: *Communicating*

	Achievement Level Descriptor (MYP3)	Task Specific Descriptor
0	The student **does not** reach a standard described by any of the descriptors below.	
1-2	The student is able to: i. use **limited** mathematical language ii. use **limited forms** of mathematical representation to present information iii. *(not demonstrated at this level)* iv. communicate through lines of reasoning that are **difficult to interpret** v. *(not demonstrated at this level)*.	The student is able to: i. **use** a minimal amount of mathematical vocabulary or algebraic notation ii. **use** one of the following effectively, but with errors: diagrams, tables, calculations, written explanation, and algebraic formula iii. *(not demonstrated at this level)* iv. **present** arguments that are difficult to interpret v. *(not demonstrated at this level)*
3-4	The student is able to: i. use **some appropriate** mathematical language ii. use **appropriate forms** of mathematical representation to present information **adequately** iii. *(not demonstrated at this level)* iv. communicate through lines of reasoning that are **able to be understood**, although these are **not always clear** v. **adequately organize** information using a logical structure.	The student is able to: i. **use** some appropriate mathematical vocabulary or algebraic notation ii. **use** two of the following effectively, but with minor errors figures, diagrams, calculations, written explanation, and algebraic formula iii. *(not demonstrated at this level)* iv. **present** arguments that can generally be understood, however are not always clear v. **show** working out that is somewhat organized using some form of logical structure.
5-6	The student is able to: i. **usually** use **appropriate** mathematical language ii. **usually** use **appropriate forms** of mathematical representation to present information **correctly** iii. move between different forms of mathematical representation **with some success** iv. communicate through lines of reasoning that are clear although **not always coherent or complete** v. **present** work that is **usually organized** using a logical structure.	The student is able to: i. usually **use** appropriate mathematical vocabulary or algebraic notation ii. **use** at least three of the following effectively: diagrams, tables, calculations, written explanation, and algebraic formula iii. **use** the different form of representation in a way so that they sometimes reinforce each other iv. **present** arguments that are clear to read, but not always coherent or complete v. **show** working out that is usually organized using a logical structure.
7-8	The student is able to: i. **consistently** use **appropriate** mathematical language ii. use **appropriate forms** of mathematical representation to **consistently** present information **correctly** iii. move **effectively** between different forms of mathematical representation iv. communicate through lines of reasoning that are **complete and coherent** v. present work that is **consistently organized** using a logical structure.	The student is able to: i. consistently **use** appropriate mathematical vocabulary or algebraic notation ii. **use** at least four of the following effectively throughout the investigation: diagrams, tables, calculations, written explanation, and algebraic formula iii. **use** the different form of representation in a way so that they effectively reinforce each other iv. **present** clear arguments that are consistently complete and coherent v. **show** working out that is consistently organized using a logical structure

Assessment Criterion D: *Applying mathematics in real-life contexts*

	Achievement Level Descriptor (MYP3)	Task Specific Descriptor
0	The student **does not** reach a standard described by any of the descriptors below.	
1-2	The student is able to: i. identify **some** of the elements of the authentic real-life situation ii. apply mathematical strategies to **find a solution** to the authentic real-life situation, **with limited success** iii. *(not demonstrated at this level)* iv. *(not demonstrated at this level)* v. *(not demonstrated at this level)*.	The student is able to: i. **identify** parts of the square cover that are not necessary to fully cover the wishing well (Q1) ii. **show** that the area of the square cover is 4 m^2 (Q1) iii. *(not demonstrated at this level)* iv. *(not demonstrated at this level)* v. *(not demonstrated at this level)*
3-4	The student is able to: i. identify the **relevant** elements of the authentic real-life situation ii. select, **with some success, adequate** mathematical strategies to model the authentic real-life situation iii. apply mathematical strategies to **reach a solution** to the authentic real-life situation iv. *(not demonstrated at this level)* v. **describe** whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: i. **use** a color of their own to **identify** the required squares in the 20x20 square grid and **identify** the required values (Q2) ii. **use** the appropriate mathematical formulas to calculate the required areas (Q2) (Q3) iii. **calculate** the area of a small square (Q2) and **calculate** the total area of the cover (Q3) iv. *(not demonstrated at this level)* v. briefly **describe** what they notice by answering both questions (Q4)
5-6	The student is able to: i. identify the **relevant** elements of the authentic real-life situation ii. select **adequate** mathematical strategies to model the authentic real-life situation iii. apply the selected mathematical strategies to **reach a valid solution** to the authentic real-life situation iv. **describe** the degree of accuracy of the solution v. **discuss** whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: i. **identify** the reason why the width of each strip is 0.5 meter (Q5) and **identify** the required lengths (Q6) ii. **apply** Pythagoras' theorem (Q6) and the appropriate area formula(s) (Q7) to calculate/find the required values iii. **calculate** the side length of the longer leg of the triangle (Q6) and **use** their findings to **find** the area of the cover (Q7) iv. briefly **describe** the accuracy of their calculations and how their final results compare to previous results (Q8) v. **discuss** whether they think that the 4-strip cover would be realistic for our circular wishing well (Q9)
7-8	The student is able to: i. identify the **relevant** elements of the authentic real-life situation ii. select **appropriate** mathematical strategies to model the authentic real-life situation iii. apply the selected mathematical strategies to **reach a correct solution** iv. **explain** the degree of accuracy of the solution v. **explain** whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: i. **identify** the width of each strip for both covers (Q10a), **draw** the required right-angled triangles (Q10b) and **identify** the required side lengths (Q10b) ii. **apply** the appropriate methods and area formula(s) to calculate/find the required values (Q10c) (Q10d) iii. **calculate** the side length of the missing legs of the triangles (Q10c) and **use** their findings to **find** the area of both covers (Q10d) iv. **write down** the required area and briefly **explain** why the calculated areas become close as the strip numbers increase (Q11) v. **state** and **explain** the required questions (Q12)

Introduction


Figure 1: Wishing well

Wishing wells, and wells in general, are structures created by digging or drilling deep into the ground in order to access water. Their shape on the ground surface is usually in the shape of a circle, as shown.

In this assessment task, we use mathematics to explore the amount (i.e. area) of material required to **cover a circular wishing well** in order to prevent things from falling inside. Let's explore!

Tasks

Our Aim
To create a cover for our circular wishing well using only rectangular pieces with as little extra material as possible.

In order to approach our task, we assume that the wishing well is perfectly circular with a radius of 1 meter.

Note that our circular wishing well may be fully covered by a square, as shown.

(1) **Identify** parts of the square cover that are *not necessary* to fully cover the circular wishing well and **show** that the area of the square cover is 4 m^2.

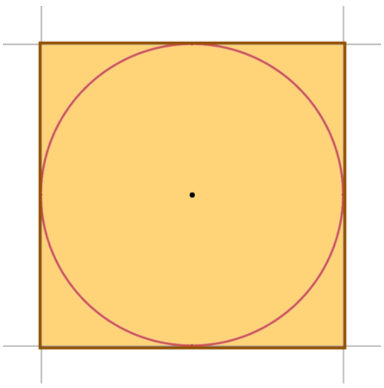


Figure 2

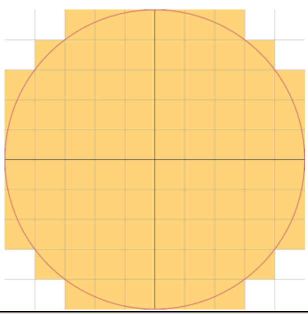
The four corners of the square are not necessary.

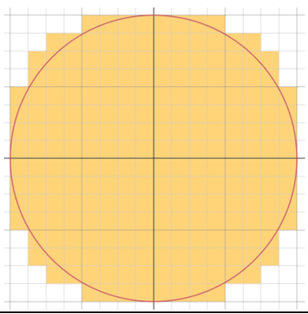
As the radius of the circle is 1 meter, its diameter must be 2 meters. This is the same as the side length of the square, which means the area of the square must be $2 \times 2 = 4$ square meters.

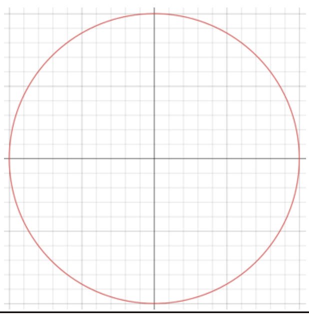
[D: 1-2, i-ii]

As there are parts of the square cover that are unnecessary for covering the circular wishing well, we know that the *area* required to cover the wishing well can be smaller than 4 m^2.

By placing finer and finer square grids on our circular wishing well, we can find a better and better way to cover our wishing well with as little extra material as possible. Below are three possible square grids, each finer then the previous one.


10x10 Square Grid


16x16 Square Grid


20x20 Square Grid

The shadings in the 10x10 and the 16x16 square grids indicate the squares that must be part of the cover in order to fully cover the wishing well.

(2) **Use** a color of your own to **identify** the squares in the 20x20 square grid that must be part of the cover in order to fully cover the wishing well. Then, in each of the three square grids shown above, **identify**

a. the *side length* and

b. the *total* number of small squares that are fully inside the circle.

Then, **calculate** the *area* of a small square and **summarize** your findings in *Table 1* below.

	Square Grids		
	10x10	**16x16**	**20x20**
Side Length of a Square *(meter)*	0.2	0.125	0.1
Area of a Square *(square meter)*	0.04	0.015625	0.01
Total Number of Small Squares	88	224	344

Table 1

[D: 3-4, i-iii]

(3) In each of the three square grids shown above, **use** your findings in question (2) to **calculate** the *total area of the cover* required to fully cover our wishing well, **showing** your work in details. **Summarize** your findings in *Table 2* below.

For the 10x10 square grid:

Area: $0.04 \times 88 = 3.52$

For the 16x16 square grid:

Area: $0.015625 \times 224 = 3.5$

For the 20x20 square grid:

Area: $0.01 \times 344 = 3.44$

	Square Grids		
	10x10	**16x16**	**20x20**
Total Area of the Cover	3.52	3.5	3.44

Table 2

[D: 3-4, ii-iii]

(4) Briefly **describe** what you notice in your calculations:

a. We notice that as the square grids become finer and finer, the area of the cover required becomes smaller and smaller. What value do you think the calculated areas approach as the square grids become finer? Why do you think so?

The calculated areas, as the square grids become finer and finer, approach the true area of the circle. We know it's $\pi r^2 = \pi(1)^2 = \pi \approx 3.14$.

b. What size square grid do you think would be *appropriate* and *realistic* to use to estimate the area of the cover required to fully cover our wishing well? Why do you think so?

Check student's response.

[D: 3-4, v]

Next, let us see if we can use another approach to cover our wishing well with as little extra material as possible: instead of placing a grid over the circle, we place strips of equal widths, an example of which is shown with 4 strips.

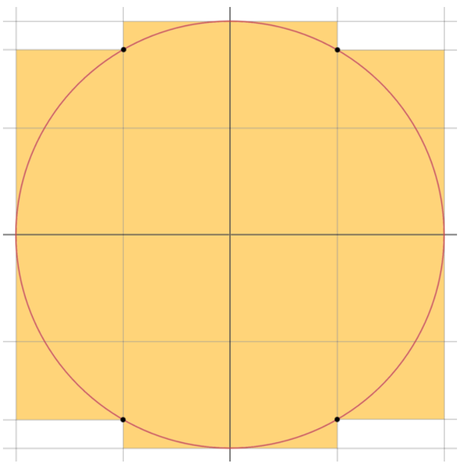


Figure 3

The question then is: how wide and how long are these strips?

As the width of each strip is the same, we can easily find the *width* of each of these strips.

However, in order to find the *length* of the strips, we must recognize/draw a right-angled triangle and use Pythagoras' theorem to find the missing side length.

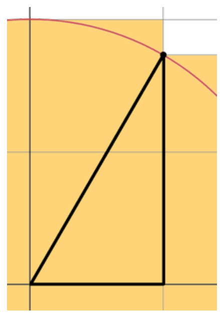


Figure 4

(5) Given that the cover is made of 4 strips, as shown in *Figure 3* above, **identify** the reason why the width of each strip is 0.5 meter.

The width is 0.5 meter because the 1-meter radius is divided into two equal parts (or the 2-meter diameter is divided into four equal parts).

[D: 5-6, i]

(6) **Identify** the length of the hypotenuse and the length of the shorter leg of the right-angled triangle drawn in *Figure 4* above. Then, **apply** a mathematical formula to **calculate** the side length of the longer leg of the triangle.

The hypotenuse of the triangle is the radius of the circle, which is 1 meter. As the width of a strip is 0.5 meter, this is the side length of the shorter leg of the triangle. The Pythagoras' theorem to solve is

$$0.5^2 + b^2 = 1^2$$

Solving this for the unknown results in

$$b^2 = 1^2 - 0.5^2 \Rightarrow b = \sqrt{}$$

[D: 5-6, i-iii]

(7) **Use** your findings above to **find** the area of the cover if it is made of 4 strips.

The area of each inner strip is $0.5 \times 2 = 1$ square meter. As there are two of these, this combined area is 2 square meters.

The area of each outer strip is $(2 \times 0.866025) \times 0.5 = 0.866025$. As there are two of these, this combined area is 1.732050 square meters.

Therefore, the area of the circular cover if it is made of 4 strips is 3.732050 square meters.

[D: 5-6, i-iii]

(8) Briefly **describe** how accurate you think your calculations are <u>and</u> how your final result (the calculated area of the circular cover) compares to your results in question (3).

Check student's response about accuracy and comparison.

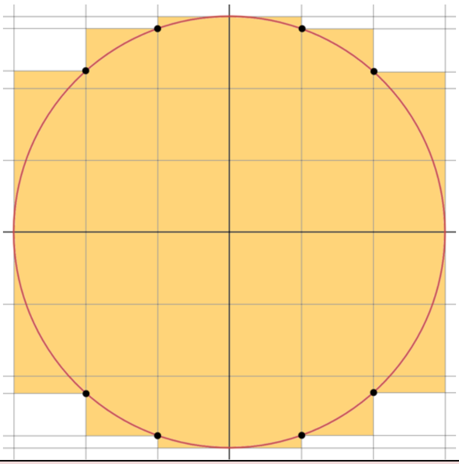
[D: 5-6, iv]

(9) **Discuss** whether you think that making a cover out of 4 strips as shown would be realistic for a circular wishing well.

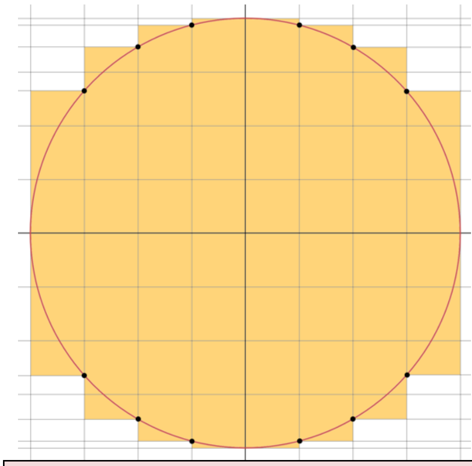
Check student's response.

[D: 5-6, v]

Next, let us place thinner strips on our circular cover. However, in order to have as little extra material as possible, let us place 6 and then 8 circular strips on the cover, as shown.



6 Strips



8 Strips

(10)For each suggested cover (the 6-strip and the 8-strip cover),

a. **identify** the width of each strip,

b. **draw** and **identify** the side lengths of the right-angled triangles required to find the length of each strip,

[D: 7-8, i]

c. **use** a method of your choice to **calculate** the side length of the missing longer leg of the right-angled triangle,

d. **use** your findings above to **find** the area of the circular cover.

[D: 7-8, ii-iii]

(11)**Write down** the area of the circular cover, if it was made of a perfect circle. Then, briefly **explain** why the calculated areas for the cover become closer to the area of the circle as the number of strips is increased.

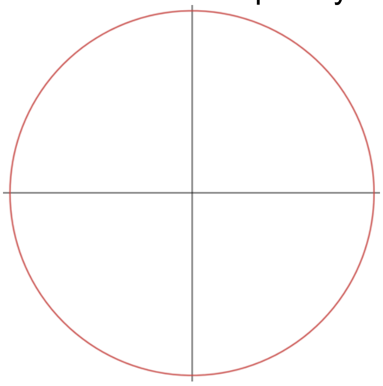
[D: 7-8, iv]

(12)**State** and **explain** why you think it would make sense to have the cover for the circular wishing well made of 8 strips instead 6 strips. Then, **suggest** a number of strips that you would *personally* prefer for the cover or our wishing well and briefly **explain** why.

[D: 7-8, v]

Extra Challenge

Use the circle shown below to **show** the number of strips you suggested in question (12). Then, **solve** each part of question (10) for the number of strips of your choice.



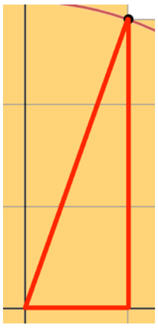
Use the space below to answer questions (10) – (12) and the Extra Challenge.

6-strip cover:

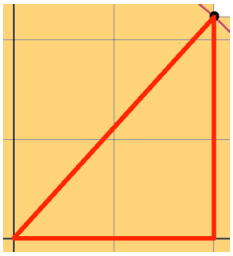
(10)(a)

The width of each strip is 1/3 meter.

(10)(b)



Shorter leg: 1/3 meter
Hypotenuse: 1 meter



Shorter leg: 2/3 meter
Hypotenuse: 1 meter

(10)(c)

For the first triangle (with the 1/3-meter shorter leg), using Pythagoras' theorem, we have

$$\left(\frac{1}{3}\right)^2 + b^2 = 1^2 \Longrightarrow b^2 = 1 - \frac{1}{9} \Longrightarrow b = \sqrt{}$$

For the second triangle (with the 2/3-meter shorter leg), using Pythagoras' theorem, we have

$$\left(\frac{2}{3}\right)^2 + b^2 = 1^2 \Longrightarrow b^2 = 1 - \frac{4}{9} \Longrightarrow b = \sqrt{}$$

(10)(d)

Starting from the outer strips, the areas are:

- $\frac{1}{3} \times (2 \times 0.745355 \ldots) = 0.4969 \ldots$
- $\frac{1}{3} \times (2 \times 0.9428 \ldots) = 0.6285 \ldots$
- $\frac{1}{3} \times (2 \times 1) = 0.66666 \ldots$

As there are two of each of these strips, the total area of the cover made from 6 strips is

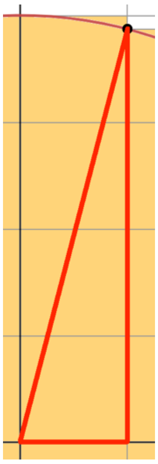
$$2 \times (0.4969 \ldots + 0.6285 \ldots + 0.6666 \ldots) = 3.584 \ldots m^2$$

8-strip cover:

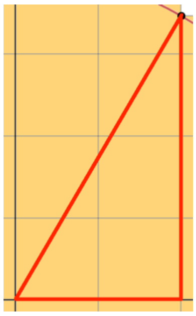
(10)(a)

The width of each strip is 1/4 or 0.25 meter.

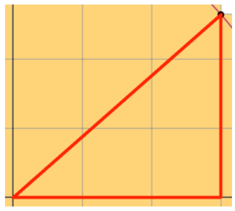
(10)(b)



Shorter leg: 1/4 meter
Hypotenuse: 1 meter



Shorter leg: 2/4 meter
Hypotenuse: 1 meter



Shorter leg: 3/4 meter
Hypotenuse: 1 meter

(10)(c)

For the first triangle (with the 1/4-meter shorter leg), using Pythagoras' theorem, we have

$$\left(\frac{1}{4}\right)^2 + b^2 = 1^2 \Rightarrow b^2 = 1 - \frac{1}{16} \Rightarrow b = \sqrt{\quad}$$

For the second triangle (with the 2/4-meter shorter leg), using Pythagoras' theorem, we have

$$\left(\frac{2}{4}\right)^2 + b^2 = 1^2 \Rightarrow b^2 = 1 - \frac{4}{16} \Rightarrow b = \sqrt{\quad}$$

For the third triangle (with the 3/4-meter shorter leg), using Pythagoras' theorem, we have

$$\left(\frac{3}{4}\right)^2 + b^2 = 1^2 \Rightarrow b^2 = 1 - \frac{9}{16} \Rightarrow b = \sqrt{\quad}$$

(10)(d)

Starting from the outer strips, the areas are:

- $\frac{1}{4} \times (2 \times 0.968245 \ldots) = 0.484122 \ldots$
- $\frac{1}{4} \times (2 \times 0.866025 \ldots) = 0.433012 \ldots$
- $\frac{1}{4} \times (2 \times 0.661437 \ldots) = 0.330718 \ldots$
- $\frac{1}{4} \times (2 \times 1) = 0.5$

As there are two of each of these strips, the total area of the cover made from 8 strips is

$$2 \times (0.4841 \ldots + 0.4330 \ldots + 0.3307 \ldots + 0.5) = 3.4956 \ldots m^2$$

(11)
The area of the circle is πr^2, which in this case, given that the radius of our wishing well is 1 meter, $\pi(1)^2 = \pi \approx 3.1415$ ... square meters.

As the number of strips increase, the area of our cover (made of strips) would get closer and closer to this area as the extra material (the parts that are not part of the circle) would become smaller and smaller.

(12)
Covering the wishing well with an 8-strip cover instead of a 6-strip cover means the area of the cover would be smaller, i.e. closer to the area of the circle.

Check student's answer for their suggestion and its explanation.