Number of Factors

Assessment Type	**Recommended Grade Level**	**MYP Criterion Level**
Mathematical Investigation	Grade 9 Standard (MYP4S)	MYP 3

MYP Assessment Criteria
Criterion B: Investigating patterns

MYP Command Terms Used
list, summarize, describe, use, predict explain, write down, suggest, find, verify, apply, justify

MYP Global Context
Identities and relationships

MYP Key Concepts
Relationships, Logic

MYP Related Concepts
Generalization, Patterns

MYP Branch of Mathematics
Numerical and abstract reasoning

MYP Topics and Skills

- Factors of numbers
- Prime numbers and factors
- Operating with algebraic expression
- Exponents
- Find, verify, and justify rules

Prior Knowledge Needed

- Finding prime factors
- Using exponents to write prime factors of numbers in power form

Assessment Description

In this assessment, students explore factors of powers of prime numbers (2, 3, 5, etc.) and are guided through a systematic process to find a pattern for the *number* of factors that powers of prime numbers have. Then, using their findings, they investigate the number of factors of any positive natural number and discover, apply, and justify a method that allows listing all factors of any given number.

Materials Needed

Pen, pencil, scrap paper, etc.

Task-specific instructions / Recommendations

The use of calculator is not required.

Assessment Criterion B: *Investigating patterns*

Level	Achievement Level Descriptor (MYP3)	Task Specific Descriptor
0	The student **does not** reach a standard described by any of the descriptors below.	
1-2	The student is able to: i. **apply**, **with teacher support**, mathematical problem-solving techniques to recognize **simple patterns** ii. **state** predictions consistent with simple patterns iii. *(not demonstrated at this level)*.	The student is able to: i. **list** the required factors (Q1), **summarize** findings (Q2), and **describe** what they notice (Q3) ii. **use** their previous findings to **predict** the number of factors and briefly explain why they think so (Q4) iii. *(not demonstrated at this level)*.
3-4	The student is able to: i. **apply** mathematical problem-solving techniques to discover **simple patterns** ii. **suggest** relationships and/or general rules consistent with **findings** iii. *(not demonstrated at this level)*.	The student is able to: i. **write down** the given numbers in power form and **list** their factors (Q5), (Q7), and **summarize** their findings (Q6), (Q8) ii. **suggest** a general rule or a mathematical formula for finding the number of factors or p^n and **list** all of its factors in terms of p and n (Q9) iii. *(not demonstrated at this level)*.
5-6	The student is able to: i. **select** and **apply** mathematical problem-solving techniques to discover **complex patterns** ii. **describe patterns** as relationships and/or general rules consistent with **findings** iii. **verify** these relationships and/or general rules.	The student is able to: i. **write down** the given numbers as products of primes using exponents/powers (Q10), **write down** the missing expressions and **find** the missing products (Q11), and **use** a method of your own to **list** *some* of the factors of 28, 45, and 72 (Q12) ii. **use** their findings to *attempt to* **describe** a general rule or a mathematical formula for finding the number of factors of $(p_1)^m \times (p_2)^n$ (Q13) iii. **verify** that their method and their general rule or general formula works (Q14)
7-8	The student is able to: i. **select** and **apply** mathematical problem-solving techniques to discover **complex patterns** ii. **describe patterns** as relationships and/or general rules consistent with **correct findings** iii. **verify** and **justify** these relationships and/or general rules.	The student is able to: i. **write down** the given numbers as products of primes using exponents/powers (Q10), **write down** the missing expressions and **find** the missing products (Q11), and **use** a method of your own to **list** *all* of the factors of 28, 45, and 72 (Q12) ii. **use** their findings to *accurately* **describe** a general rule or a mathematical formula for finding the number of factors of $(p_1)^m \times (p_2)^n$ (Q13) iii. *correctly* **verify** that their method and their general rule or general formula works (Q14) and **justify** why their method ensures that all factors are found and **explain** why the same method might be difficult to apply (Q15)

Introduction

The factors of a natural number are the natural numbers which divide exactly into it.
For example,

- $24 \div 4 = 6$, which means 4 is a factor of 24,
- $24 \div 6 = 4$, which means 6 is a factor of 24,
- $24 \div 8 = 3$, which means 8 is a factor of 24,
- $24 \div 5 = 4$, remainder 4, which means 5 is not a factor of 24.

There are several methods that can be used to find the factors of a number, but is there a way to know whether we found them all or we missed some of them when we were listing them? Is there a way to know *the number of factors* that a number has? Let's explore!

Tasks

The factors of 2 are 1 and 2. The factors of 4 are 1, 2, and 4, as shown below.

(1) For each number given below, **list** its factors in ascending order.

a. 2

1, 2

b. 4

1, 2, 4

c. 8

1, 2, 4, 8

d. 16

1, 2, 4, 8, 16

e. 32

1, 2, 4, 8, 16, 32

f. 64

1, 2, 4, 8, 16, 32, 64

[B: 1-2, i]

(2) **Summarize** your findings in the table provided below.

Number (ordinary form)	***Number (power form)***	***List of factors (ordinary form)***	***Number of factors***
2	2^1	1, 2	2
4	2^2	1, 2, 4	3
8	2^3	1, 2, 4, 8	4
16	2^4	1, 2, 4, 8, 16	5
32	2^5	1, 2, 4, 8, 16, 32	6
64	2^6	1, 2, 4, 8, 16, 32, 64	7

Table 1

[B: 1-2, i]

(3) After taking a closer look at the numbers in power form (2^{nd} column from the left) and the number of their factors (first column from the right), briefly **describe** what you notice.

The exponent/power of the number in power form is one less than the number of factors the number has. Or, the other way around, the number of factors of a number is 1 more than the exponent/power of the prime number 2.

[B: 1-2, i]

(4) **Use** your findings above and your description in question (5) to **predict** the number of factors of each of the numbers given below. Briefly **explain** why you think so.

a. 64

$64 = 2^6$

This means it has 7 factors.

b. 128

$128 = 2^7$

This means it has 8 factors

[B: 1-2, ii]

(5) For each number given below,

a. **write down** the number in power form, and

b. **list** its factors in ascending order.

i. $3 = 3^1$

1, 3

ii. $9 =$ __3^2__

1, 3, 9

iii. $27 =$ __3^3__

1, 3, 9, 27

iv. $81 =$ __3^4__

1, 3, 9, 27, 81

v. $243 =$ __3^5__

1, 3, 9, 27, 81, 243

vi. $729 =$ __3^6__

1, 3, 9, 27, 81, 243, 729

[B: 3-4, i]

(6) Similarly to question (2), **summarize** your findings in the table provided below.

Number (ordinary form)	***Number (power form)***	***List of factors (ordinary form)***	***List of factors (power form)***	***Number of factors***
3	3^1	1, 3	$3^0, 3^1$	2
9	3^2	1, 3, 9	$3^0, 3^1, 3^2$	3
27	3^3	1, 2, 4, 8	$3^0, 3^1, 3^2, 3^3$	4
81	3^4	1, 2, 4, 8, 16	$3^0, 3^1, 3^2, 3^3, 3^4$	5
243	3^5	1, 2, 4, 8, 16, 32	$3^0, 3^1, 3^2, 3^3, 3^4, 3^5$	6
729	3^6	1, 2, 4, 8, 16, 32, 64	$3^0, 3^1, 3^2, 3^3, 3^4, 3^5, 3^6$	7

Table 2

[B: 3-4, i]

(7) Similarly to question (5), for each number given below,
   a. **write down** the number in power form, and
   b. **list** its factors in ascending order.

i. $5 = 5^1$

1, 5

ii. $25 = \_\_5^2\_\_$

1, 5, 25

iii. $125 = \_\_5^3\_\_$

1, 5, 25, 125

iv. $625 = \_\_5^4\_\_$

1, 5, 25, 125, 625

v. $3125 = \_\_5^5\_\_$

1, 5, 25, 125, 625, 3125

vi. $15625 = \_\_5^6\_\_$

1, 5, 25, 125, 625, 3125, 15625

[B: 3-4, i]

(8) Similarly to question (6), **summarize** your findings in the table provided below.

Number (ordinary form)	***Number (power form)***	***List of factors (ordinary form)***	***List of factors (power form)***	***Number of factors***
5	5^1	1, 5	5^0, 5^1	2
25	5^2	1, 5, 25	5^0, 5^1, 5^2	3
125	5^3	1, 5, 25, 125	5^0, 5^1, 5^2, 5^3	4
625	5^4	1, 5, 25, 125, 625	5^0, 5^1, 5^2, 5^3, 5^4	5
3125	5^5	1, 5, 25, 125, 625, 3125	5^0, 5^1, 5^2, 5^3, 5^4, 5^5	6
15625	5^6	1, 5, 25, 125, 625, 3125, 15625	5^0, 5^1, 5^2, 5^3, 5^4, 5^5, 5^6	7

Table 3

[B: 3-4, i]

Let p represent a *prime number* (such as 2, 3, 5, 7, and so on) and let n represent a *positive natural number* (such as 1, 2, 3, 4, and so on).

(9) Based on your findings, **suggest** a general rule or a mathematical formula for finding the number of factors of p^n. Then, **list** all of its factors in terms of p and n.

When the prime number p is raised to a positive natural number power n, it has $n + 1$ factors. These factors are

$$p^0, p^1, p^2, p^3, \dots, p^n$$

[B: 3-4, ii]

Next, let's explore positive natural numbers that are products of two prime numbers.

(10)**Write down** each of the numbers below as a product of primes **using** powers/exponents for the primes when necessary.

a. $20 = \_2 \times 2 \times 5 = 2^2 \times 5\_$

c. $45 = \_3 \times 3 \times 5 = 3^2 \times 5\_$

b. $28 = \_2 \times 2 \times 7 = 2^2 \times 7\_$

d. $72 = \_2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2\_$

[B: 5-8, i]

In order to systematically find all factors, we may use a table similar to *Table 4* shown below. This helps us ensure that no factors are missing from our list of factors.

(11)**Write down** the missing expressions and **find** the missing products in *Table 4* below.

20		Powers of 2		
		$\mathbf{2^0}$	$\mathbf{2^1}$	$\mathbf{2^2}$
Powers of 5	$\mathbf{5^0}$	$2^0 \times 5^0 = 1$	$2^1 \times 5^0 = 2$	$2^2 \times 5^0 = 4$
	$\mathbf{5^1}$	$2^0 \times 5^1 = 5$	$2^1 \times 5^1 = 10$	$2^2 \times 5^1 = 20$

Table 4

[B: 5-8, i]

(12)**Use** a method of your own, which could be a table similar to *Table 4* above if you prefer, to **list** all factors of the three other numbers in question (10).

Check student's work demonstrating a systematic method of finding all factors:

- 28 has 6 factors,
- 45 has 6 factors,
- 72 has 12 factors.

[B: 5-8, i]

Let p_1 and p_2 represent two *different prime numbers* (such as 2, 3, 5, 7, and so on) and let m and n represent *positive natural numbers* (such as 1, 2, 3, 4, and so on).

(13) **Use** your findings above to **describe** a general rule or mathematical formula for finding the number of factors of $(p_1)^m \times (p_2)^n$.

The factors of $(p_1)^m$ are $(p_1)^0$, $(p_1)^1$, $(p_1)^2$, ..., $(p_1)^m$. Similarly, the factors of $(p_2)^n$ are $(p_2)^0$, $(p_2)^1$, $(p_2)^2$, ..., $(p_2)^n$. One of these would be the rows and the other would be the columns of a table.

Therefore, there would be $(m+1) \times (n+1)$ factors listed in that table, which would be all factors, without missing any.

[B: 5-8, ii]

(14) **Verify** that your method and general rule or mathematical formula works

a. by **applying** the method you used in question (12) to **list** all factors of 675, and

b. by **applying** the general rule or mathematical formula you described in question (13) to **find** the number of factors of 675.

a)

$$675 = 3 \times 3 \times 3 \times 5 \times 5 = 3^3 \times 5^2$$

675		Powers of 3			
		3^0	3^1	3^2	3^3
Powers of 5	5^0	$5^0 \times 3^0 = 1$	$5^0 \times 3^1 = 3$	$5^0 \times 3^2 = 9$	$5^0 \times 3^3 = 27$
	5^1	$5^1 \times 3^0 = 5$	$5^1 \times 3^1 = 15$	$5^1 \times 3^2 = 45$	$5^1 \times 3^3 = 135$
	5^2	$5^2 \times 3^0 = 25$	$5^2 \times 3^1 = 75$	$5^2 \times 3^2 = 225$	$5^2 \times 3^3 = 675$

There are 12 factors, as shown in the table above.

b)

Given that $675 = 3^3 \times 5^2$, the number of factors of 675 is $(3+1) \times (2+1) = 4 \times 3 = 12$.

[B: 5-8, iii]

(15) Briefly **justify** why the method shown in question (11) ensures that all factors are found. Then, briefly **explain** why that same method might be difficult to apply for a number with three or more *different* prime factors, such as $2520 = 2^3 \times 3^2 \times 5 \times 7$.

Using the "table method" ensures that each prime factor (and their powers) is multiplied by each of the other prime factors (and their powers).

If there are three or more different prime factors, we can't create a table the same way as we'd need to have a 3rd dimension for the 3rd different prime factor and the repeated tables for the rest. After a while, this method gets too complicated.

[B: 7-8, iii]